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GROUP {1, −1, i, −i} CORDIAL LABELING OF SOME SPLITTING GRAPHS

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ABSTRACT

Let G be a (p,q)graph and A be a group. Let f : V (G) → A be a function. The order of a ∈ A is the least positive integer n such that $a^N = e$. We denote the order of a by o(a). For each edge uv assign the label 1 if (o(f (u)), o(f (v))) = 1or 0 otherwise.
f is called a group A Cordial labeling if $|v_F (a) - v_F (b)| \leq 1$ and $|e_F (0) - e_F (1)| \leq 1$, where $v_F (x)$ and $e_F (n)$ respectively denote the number of vertices labeled with an element x and number of edges labeled with n(n = 0, 1). A graph which admits a group A Cordial labeling is called a group A Cordial graph. The Splitting graph of G, $S^{'}(G)$ is obtained from G by adding for each vertex v of G, a new vertex $v^{'}$ so that $v^{'}$ is adjacent to every vertex that is adjacent to v. Note that if G is a (p, q)

graph then $S^{'}(G)$ is a (2p, 3q) graph. In this paper we prove that Splitting graphs of Path P_N , Cycle C_N and Wheel W_N are group {1, −1, i, −i} Cordial for evey n

Keywords: Cordial labeling, group A Cordial labeling, group {1, −1, i, −i} Cordial labeling, splitting graph .

AMS subject classification: 05C78

I. INTRODUCTION

Graphs considered here are finite, undirected and simple. Let A be a group. The order of a ∈ A is the least positive integer n such that $a^N = e$. We denote the order of a by o(a). Cahit [3] introduced the concept of Cordial labeling.

Motivated by this, we defined group A cordial labeling and investigated some of its properties. We also defined group {1, −1, i, −i} cordial labeling and dis-cussed that labeling for some standard graphs [1 & 2] . The Splitting graph of G, $S^{'}(G)$ is obtained from G by adding for each vertex v of G, a new vertex $v^{'}$ so that $v^{'}$ is adjacent to every vertex that is adjacent to v. Note that if G is a (p, q) graph then $S^{'}(G)$ is a (2p, 3q) graph. In this paper we discuss the labeling for Splitting graphs of some graphs. Terms not defined here are used in the sense of Harary[5] and Gallian [4].

The greatest common divisor of two integers m and n is denoted by (m, n) and m ad n are said to be relatively prime if (m, n) = 1. For any real number x, we denote by $\lfloor x \rfloor$, the greatest integer smaller than or equal to x and by $\lceil x \rceil$, we mean the smallest integer greater than or equal to x.

A path is an alternating sequence of vertices and edges, $v_1, e_1, v_2, e_2, ..., e_{N-1}, v_N$, which are distinct, such that e_I is an edge joining v_I and v_{I+1} for
1 $\leq i \leq n-1$. A path on n vertices is denoted by P_N . A path $v_1, e_1, v_2, e_2, ..., e_{N-1}, v_N , e_N , v_1$ is called a cycle and a cycle on n vertices is denoted by C_N .

Given two graphs G and H , G+H is the graph with vertex set V (G)∪V (H) and edge set E(G) ∪ E(H) ∪ {uv/u ∈ V (G), v ∈ V (H)}. A Wheel W_N is de-fined as $C_N + K_1$.

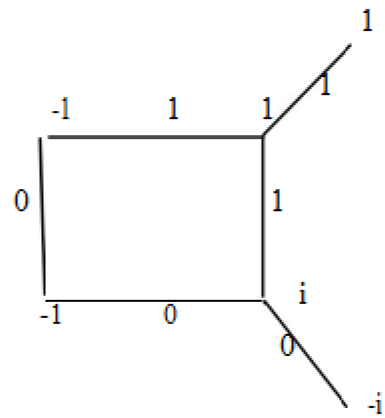

Fig 2.1

II. GROUP {1, −1, i, −i} CORDIAL GRAPHS

Definition 2.1. Let G be a (p,q)graph and consider the group

A= {1, −1, i, −i} with multiplication. Let f : V (G) → A be a funtion.

For each edge uv assign the label 1 if (o(f (u)), o(f (v))) = 1or 0 otherwise. f is called a group {1, −1, i, −i} Cordial labeling if $|v_F(a) - v_F(b)| \leq 1$ and $|e_F(0) - e_F(1)| \leq 1$, where $v_F(x)$ and $e_F(n)$ respectively denote the number of vertices labeled with an element x and number of edges labeled with n(n = 0, 1). A graph which admits a group {1, −1, i, −i} Cordial labeling is called a group {1, −1, i, −i} Cordial graph

Example 2.2. A simple example of a group {1, −1, i, −i} Cordial graph is given in Fig. 2.1.

Definition 2.3. The Splitting graph of G, $S'(G)$ is obtained from G by adding for each vertex v of G, a new vertex v' so that v' is adjacent to every vertex that is adjacent to v. Note that if G is a (p, q) graph then $S'(G)$ is a (2p, 3q) graph.

We now investigate the group {1, −1, i, −i} Cordial labeling of Splitting Graph of Path, Cycle and Wheel.

Theorem 2.4. The splitting graph of the path, $S'(P_N)(n \geq 1)$, is group {1, −1, i, −i} cordial for every n.

Proof. Let $u_1, u_2, ..., u_N$ be the vertices of the path P_N and let $v_1, v_2, ..., v_N$

be the newly added vertices. Number of vertices in $S'(P_N)$ is 2n and number of edges is $3(n - 1) = 3n - 3$.

Case(1) : n is even.

Let n = 2k, ($k \geq 1$, $k \in Z$). Each vertex label should appear k times. One edge label should appear $3k - 2$ times and another should appear $3k - 1$ times. Define a labeling f of $S'(P_N)$ as follows.

Label the vertices $v_1, u_2, u_3, ..., u_K$ with 1. Label the remaining vertices arbi-trarily so that k of them get label −1, k of them get label i and k of them get label −i. Number of edges with label $1 = 1 + (k - 1)3 = 3k - 2$.

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Case(2) : n is odd.

Let $n = 2k + 1$, $(k \geq 1, k \in Z)$. Two vertex labels should appear $k + 1$ times and two should appear k times. Each edge label should appear $3k$ times. Define a labeling f of $S'(P_N)$ as follows.

Label the vertices $v_1, v_2, u_3, u_3, u_4, \ldots, u_{K+1}$ with 1. Label the remaining vertices arbitrarily so that $k + 1$ of them get label -1, k of them get label i and k of them get label $-i$. Number of edges with label $1 = 1 + 2 + (k - 1)3 = 3k$. Table 1 shows that f is a group $\{1, -1, i, -i\}$ cordial labeling. Illustration of

Table 1

n	$v_F(1)$	$v_F(-1)$	$v_F(i)$	$v_F(-i)$	$e_F(0)$	$e_F(1)$
$2k$	k	k	k	k	$3k - 1$	$3k - 2$
$2k + 1$	$k + 1$	$k + 1$	k	k	$3k$	$3k$

the labeling for $n = 5$ is given in F ig.2.2.

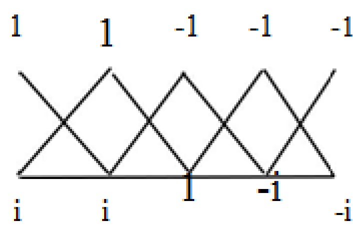

Fig. 2.2

Theorem 2.5. The splitting graph of the cycle, $S'(C_N)(n \geq 3)$ is group

$\{1, -1, i, -i\}$ cordial for every n.

Proof. Let the vertices on the cycle be labelled as $u_1, u_2, \ldots, u_N$ and let $v_1, v_2, \ldots., v_N$ be the newly added vertices so that for $1 \leq i \leq n$, v_I is adja-

cent to the neighbours of u_I . Number of vertices in $S'(C_N)$ is $2n$ and number of edges is $3n$.

Case(1) : n is even.

Let $n = 2k$, $(k \geq 2, k \in Z)$. Each vertex label should appear k times and each edge label should appear $3k$ times. Define a labeling f of $S'(C_N)$ as follows. Label the vertices $v_1, u_3, u_4, \ldots, u_{K+1}$ with 1. Label the remaining vertices ar-bitrarily so that k of them get label -1, k of them get label i and k of them get label $-i$. Number of edges with label $1 = 2 + 4 + (k - 2)3 = 3k$. Case(2) : n is odd.

Let $n = 2k + 1$, $(k \geq 1, k \in Z)$. Two vertex labels should appear $k + 1$ times and two should appear k times. One edge label should appear $6k + 1$ times and one should appear$6k + 2$ times. Define a labeling f of $S'(C_N)$ as follows.

Label the vertices v_1, v_2, u_3, u_3, u_4, ..., u_{K+1} with 1. Label the remaining vertices arbitrarily so that k + 1 of them get label −1, k of them get label i and k of them get label −i. Number of edges with label 1 = 2 + 2 + (k − 1)3 = 3k + 1. Table 2 shows that f is a group {1, −1, i, −i} cordial labeling.

Table 2

n	$v_F(1)$	$v_F(-1)$	$v_F(i)$	$v_F(-i)$	$e_F(0)$	$e_F(1)$
2k	k	k	k	k	3k	3k
2k + 1	k + 1	k + 1	k	k	3k + 2	3k + 1

Illustration of the labeling for $S'(C_5)$ is given in F ig.2.3

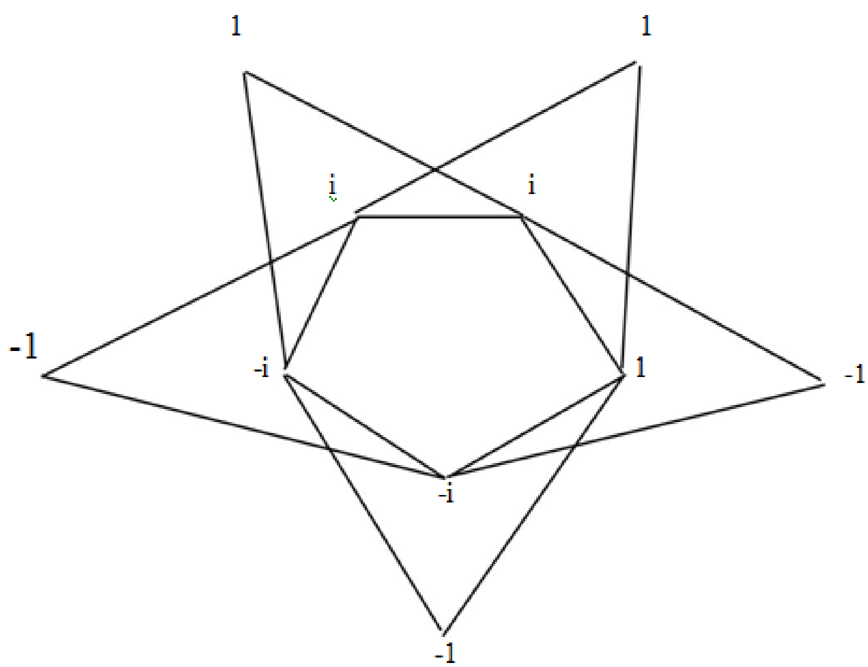

Fig 2.3

Theorem 2.6. The splitting graph of the Wheel $S'(W_N)(n \geq 3)$, is group

{1, −1, i, −i} cordial for every n.

Proof. Let the center of the Wheel be labelled as u, the corresponding ver-tex of $S'(W_N)$ by v, the vertices on the rim of the Wheel by u_1, u_2, ..., u_N in order and the corresponding vertices of $S'(W_N)$ by v_1, v_2, ..., v_N accordingly so that for $1 \leq i \leq n$, v_I is adjacent to the neighbours of u_I . Also v is adjacent to the neighbours of u. Number of vertices in $S'(W_N)$ is 2n + 2 and number of edges is 6n.

Case(1) : n is odd.

Let $n = 2k + 1$, ($k \geq 1$, $k \in Z$). Each vertex label should appear $k + 1$ times and each edge label should appear $6k + 3$ times. Define a labeling f of $S'(W_N)$ as follows.
Label the vertices $v_1, u_1, u_3, \ldots, u_{2K-1}$ with 1. Label the remaining vertices ar-bitrarily so that $k + 1$ of them get label -1, $k + 1$ of them get label i and $k + 1$ of them get label $-i$. Number of edges with label $1 = 6k + 3$.

Case(2) : n is even.

Let $n = 2k$, ($k \geq 2$, $k \in Z$). Two vertex labels should appear $k + 1$ times and twoother labels should appear k times. Each edge label appears 6k times. Define a labeling f of $S'(W_N)$ as follows.

Label the vertices $u_1, u_3, \ldots, u_{2K-1}$ with 1. Label the remaining vertices arbi-trarily so that k of them get label -1, $k + 1$ of them get label i and $k + 1$ of them get label $-i$. Number of edges with label $1 = 6k$. is a group $\{1, -1, i, -i\}$ cordial labeling.

Table 3

n	$v_F(1)$	$v_F(-1)$	$v_F(i)$	$v_F(-i)$	$e_F(0)$	$e_F(1)$
$2k + 1 (k \geq 1)$	$k + 1$	$k + 1$	$k + 1$	$k + 1$	$6k + 3$	$6k + 3$
$2k (k \geq 2)$	k	k	$k + 1$	$k + 1$	$6k$	$6k$

Illustration of the labeling for $S'(W_4)$ is given in F ig.2.4.

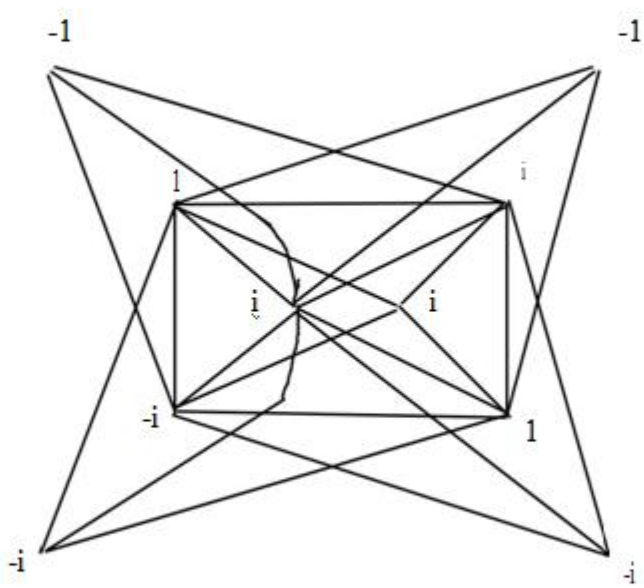

Fig. 2.4

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